# MAE 4230-5230
# Lecture 2 - Notes

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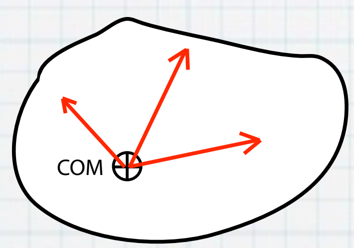
# Rigid body vs fluid

## Rigid body

* only one point is needed in order to track the dynamics of the object (e.g. COM)
* dynamics are solution to ODEs (ordinary differential equations)
* the dynamics are given by

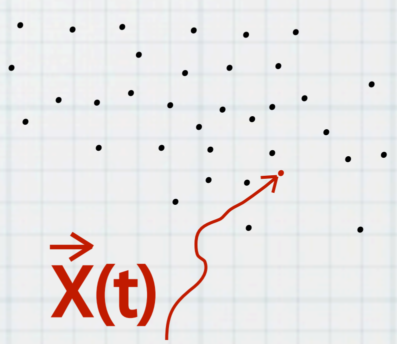



| | | | |
|---|---|---|---|
| *position* | : | $\vec{X}$ | (3) |
| *orientation* | : | $\theta,\ \phi,\ \psi$ | (3) |
| *velocity* | : | $\vec{U}_{com}$ | (3) |
| *rotational velocity* | : | $\vec{\Omega}$ | (3) |

between parentheses is the number of unknowns

## Fluid

* need to track all particles
* the fluid is a continuum medium, hence definition of fields (e.g. velocity field, temperature field, etc)
* dynamics are solution to PDEs (partial differential equations)
  * e.g. we solve for $\vec{U}(\vec{x},t)$ at every point instead of $\vec{U}_{com}(t)$



# Reynolds number

Definition: $$Re = \frac{UL}{\nu}$$

$$\begin{aligned} U &= velocity\ scale \\ L &= length\ scale \\ \nu &= kinematic\ viscosity \end{aligned}$$

Dimensions:

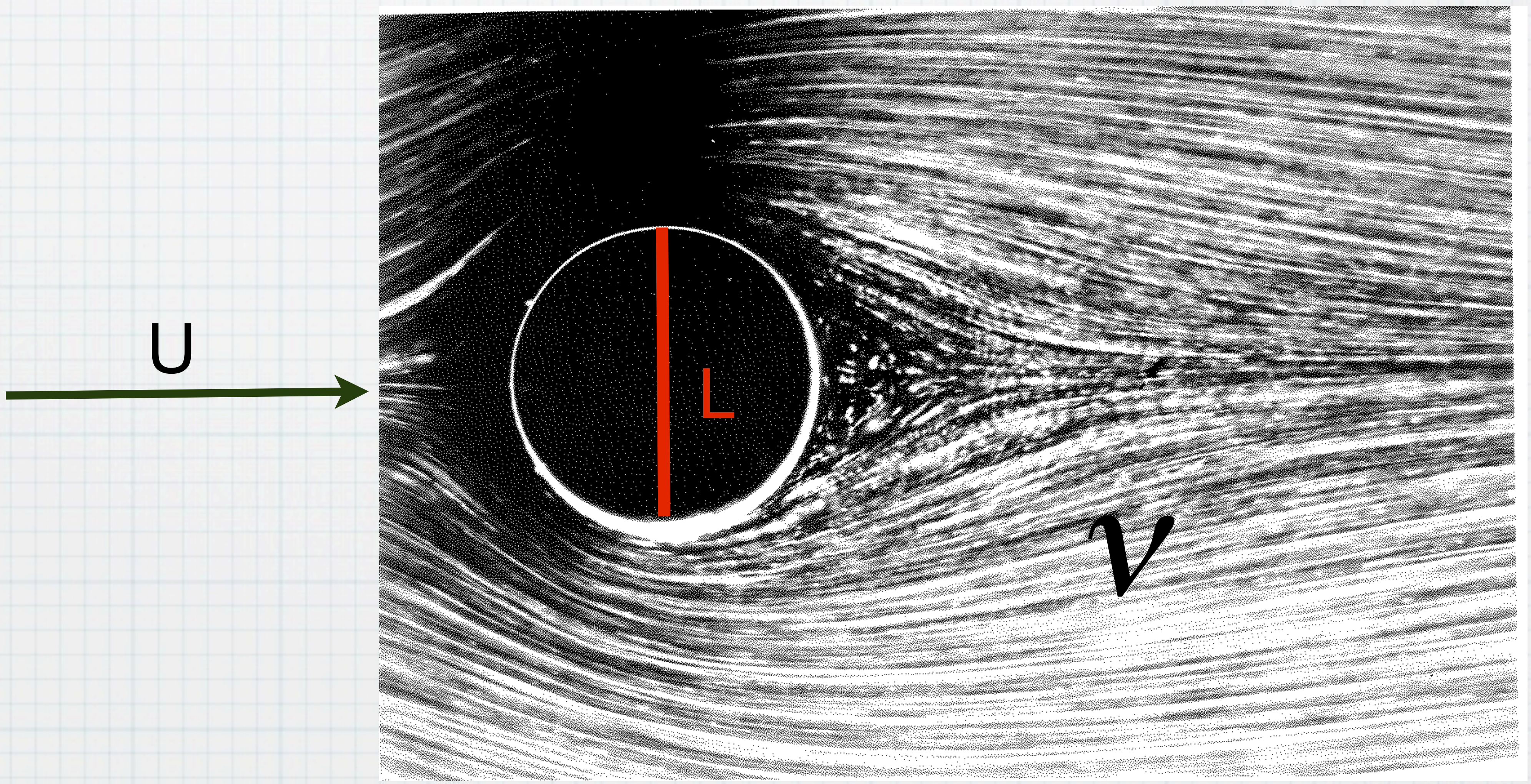
$$\begin{aligned} [U] &= m/s \\ [L] &= m \\ [\nu] &= m^2/s \end{aligned}$$

Re is dimensionless!

# Reynolds number

$$Re = \frac{UL}{\nu}$$

Illustration of parameters
for flow past a cylinder



# Reynolds number

* What are the typical values of Re number for different flow regimes?

$$Re = \frac{fluid\ inertia}{viscous\ forces}$$

* e.g. 1: Jane walking leisurely in the auditorium

$$\left.\begin{array}{l} V_{ref} \sim 1m/s \\ L_{ref} \sim 0.5\,m \\ \nu_{air} \sim 10^{-5}\,m^2/s \end{array}\right\} \quad Re \sim 5\,10^4$$

* pretty turbulent regime! If you imagine smoke behind her, you'll see swirls and eddies, signature of turbulent flow.

# Reynolds number

* What are the typical values of Re number for different flow regimes?

$$Re = \frac{fluid\ inertia}{viscous\ forces}$$

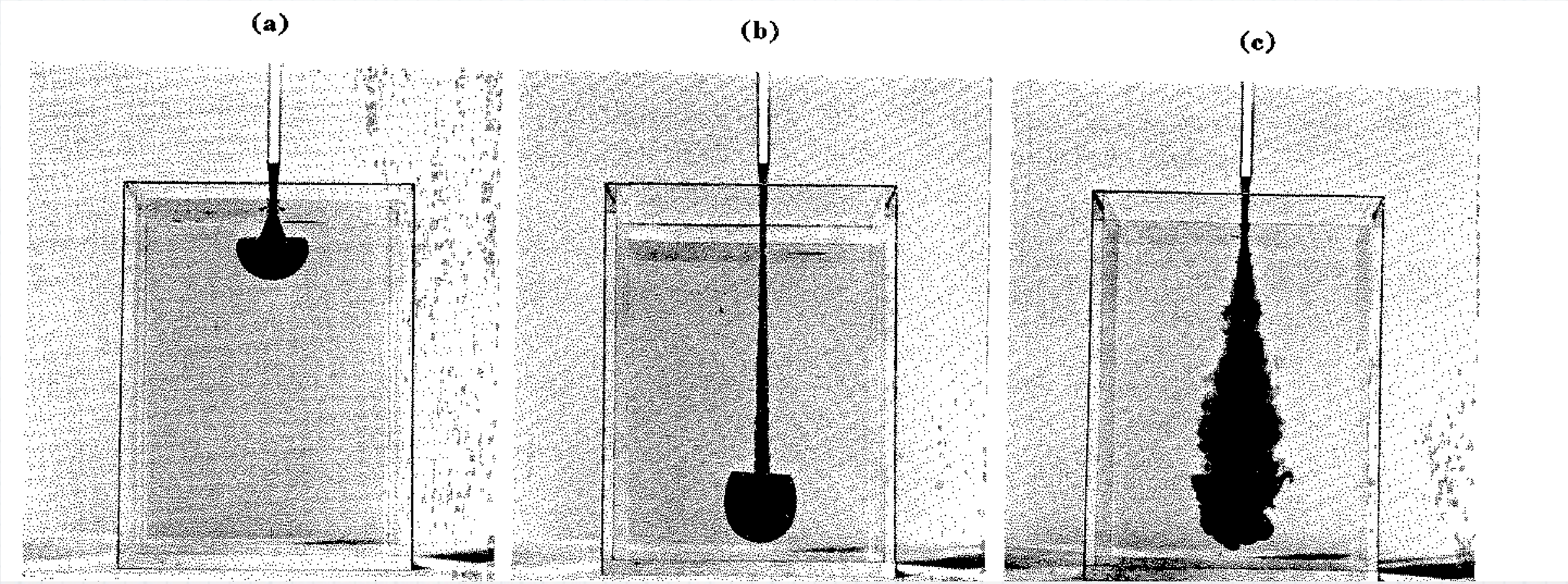
* e.g. 2: a 10µm bead moving at a speed of 15cm/s in water

$$\left.\begin{aligned} V_{ref} &= 0.15\, m/s \\ L_{ref} &= 10^{-5}\, m \\ \nu_{water} &= 10^{-6}\, m^2/s \end{aligned}\right\} \quad Re = 1.5 \sim 1$$

* a micro-bead @ high speed! This is borderline. Typical bacteria swim @ a couple body lengths per second, e.g. 20µm/s

# Which flow has a higher Reynolds number?

- 3 containers filled with different fluids
- an incoming jet of ink is injected via a pipette




$$Re_a < Re_b < Re_c$$

- visually, we can say that fluid ‘a’ is thicker or ‘stickier’ and the jet cannot push ‘hard’ enough
- fluid ‘c’ is able to penetrate further since it is not decelerated or damped by the ambient fluid

# Mental picture of the Reynolds number

## Stirring coffee *vs* Stirring honey

(* admire the art! *)

* once you stop stirring coffee, it keeps rotating. The moving fluid has enough inertia (from stirring with a spoon) to overcome the viscous forces. If you are not convinced, go to Starbucks in Collegetown, order a *Frappucino* and try it out ;-)

* once you stop stirring honey, it stops rotating immediately. The viscous forces are way higher than the inertia imparted to the honey.

# Survey questions

- List the fluid variables needed to describe its state

$$\rho, \ T, \ \vec{V}$$

- List the fluid parameters that distinguish air from water

$$\rho, \ \mu$$

- Fluid equations
  - Conservation of mass (incompressible) $\nabla . \vec{u} = 0$
  - Conservation of momentum $\rho \left[ \frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} \right] = -\nabla p + \mu \nabla^2 u$