MAE4230/5230 Fall 2011

Intermediate Fluid Dynamics

Jane Wang

Mechanical and Aerospace Engineering
Cornell University

Lecture 17

2011-04-04 M

- Streamlines, Vorticity, Circulation

- Reading: Chapter 1, Acheson

Streamline

- the tangent to the streamline // u(x,t)

- a streamline can be described by a parametric function: x(s), y(s), z(s). by definition, dx(s)/dt = ux, dy(s)/dt = uy, dz(s)/dt=uz

- examples: linear shear flow, circular flow, and also flow around a stagnation point.

Streak lines, Path lines

- streak line: a snapshot of a line written by dye particles, which are injected to the fluid continuously.
- path line: a line traced out by a fluid particle
- In steady flow, the streamline = streak line = path line, but in general, they are not equivalent

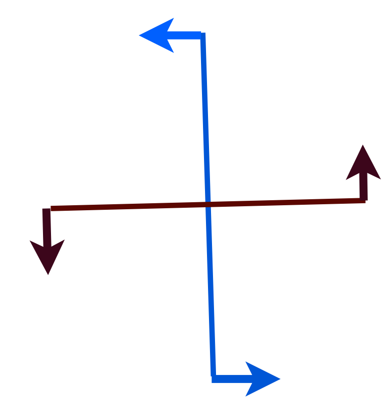
Vorticity

$$\omega = \nabla \times U$$

define vector cross product

work out vorticity in 2D

magnitude = dv/dy - du/dx
direction = normal to the 2D plane



interpretation of the vorticity field:
 the sum of the angular rotations of two infinitesimal orthogonal lines

Parallel streamlines does not imply zero vorticity

- typical drawing of a vortex is a spiral, but not all vortical flows are circular.
- consider the shear flow, the stream lines are parallel, but the vorticity is nonzero. where is the circulation in this picture? if we are a fluid particle co-moving with the flow, we see that the fluid above moves faster, and the fluid below slower. the flow is rotation around us. The shear flow velocity = constant translational velocity + rotational velocity

Circular streamlines does not imply non-zero vorticity

- Consider the flow generated by a line vortex. $u(r) = 1/r$

- to show that the vorticity associated with this flow is zero, we can first show that the circulation is constant, independent of r. now if we enclose the origin with an infinitesimally small circle, the net circulation is still finite. thus the vorticity must be concentrated at r=0. in other words, there is no vorticity outside r=0. if there is, the circulation around circles of different radii will be differently.

Vorticity and Circulation

- applying Stoke's theorem, the circulation is the same as the total vorticity inside the closed contour.

- solid body rotation $u = \Omega \times r$, we can show that $\omega = 2\Omega$ in three ways. a) it's the sum of the rotational rate of two orthogonal lines, b) circulation = total vorticity, c) direct application of the vector calculus using curl.