

MAE 4230/5230 Homework 5 Solution

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Consider developing flow in a pipe of length $L = 8$ m, diameter $D = 0.2$ m, $\rho = 1$ kg/m³, $\mu = 2 \times 10^{-3}$ kg/m s, and entrance velocity $u_{\text{in}} = 1$ m/s. This case is solved in the laminar pipe flow tutorial at <https://confluence.cornell.edu/x/6YQaBQ>. Go through the tutorial. You can download the mesh at the top of the geometry step and skip the geometry and mesh steps. Use FLUENT with the “second-order upwind” scheme for momentum to solve for the flowfield on meshes of 100×10 , 100×20 and 100×40 (axial points \times radial points).

1. Plot the axial velocity profiles at the exit obtained from the three meshes. Also, plot the corresponding velocity profile obtained from fully-developed pipe analysis. Indicate the equation you used to generate this profile. In all, you should have four curves in a single plot. Use a legend to identify the various curves. Axial velocity u should be on the abscissa and r on the ordinate.

Hint: In FLUENT, you can write out the data in any “XY” plot to a file by selecting the “Write to File” option in the Solution XY Plot menu. Then click on Write and enter a filename. You can strip the headers and footers in this file and read this into MATLAB as column data using the load function in MATLAB.

Solution: Fully-developed theory gives the classic parabolic profile (see equation below). The pressure gradient can be calculated from the volume flow rate Q . Q is constant along the pipe; From the uniform inlet profile, Q can be calculated as

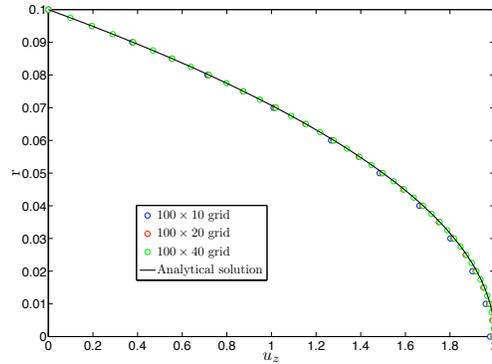
$$Q = (1) (\pi r^2)$$

The resulting equation for the exit profile is

$$u_z = 2 \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

This is compared with the FLUENT results in the figure below.

2. Calculate the shear stress τ_{rz} at the wall in the fully-developed region for the three meshes. Calculate the corresponding value from fully-developed pipe analysis. For each mesh, calculate the % error relative to the analytical value. Include your results as a table.



Solution: For fully developed flow,

$$\tau_{rz} = \frac{1}{2} \frac{dp}{dz} R = 0.08$$

Mesh	τ_{rz}	% error
100x10	0.0791892	-1.0135
100x20	0.0797997	-0.2504
100x40	0.0799599	-0.0501

3. At the exit of the pipe where the flow is fully-developed, we can define the error in the centerline velocity as

$$\epsilon = \frac{|u_c - u_{\text{exact}}|}{u_{\text{exact}}}$$

where u_c is the centerline value from FLUENT and u_{exact} is the corresponding exact (analytical) value. We expect the error to take the form

$$\epsilon = K \Delta r^p$$

where the coefficient K and power p depend upon the order of accuracy of the discretization. Using MATLAB, perform a linear least squares fit of

$$\ln \epsilon = \ln K + p \ln \Delta r$$

to obtain the coefficients p and K . Plot ϵ vs. Δr (using symbols) on a log-log plot. Add a line corresponding to the least-squares fit to this plot.

Solution: See below.

4. Let's see how p changes when using a first-order accurate discretization. In FLUENT, use "first-order upwind" scheme for momentum to solve for the flowfield on the three meshes. Repeat the calculation of coefficients p and K as above. Add this ϵ vs. Δr

data (using symbols) to the above log-log plot. Add a line corresponding to the least-squares fit to this plot. In all, you should have four curves on this plot (two each for second- and first-order discretization). Make sure you include an appropriate legend in the figure.

Contrast the value of p obtained in the two cases and briefly explain your results (2-3 sentences).

Hint: To interpret your results, you should keep in mind that the first or second-order upwind discretization applies only to the inertia (advection) terms in the momentum equation. The discretization of the viscous terms is always second-order accurate.

Solution: The table of error values is given below.

Δr	ϵ (II-order)	ϵ (I-order)
0.01	0.00990	0.00990
0.005	0.00250	0.00249
0.0025	0.00063	0.00063

The curve fit of the data is shown in the figure below. The resulting coefficients are $K = 93.28$ and $p = 1.99$ for the second-order upwind scheme. The coefficients are $K = 93.16$ and $p = 1.99$ for the first-order upwind scheme. Please note that the second-order upwind scheme takes longer to converge, especially for the 100×40 grid and the errors and hence the fitting coefficients are dependent on the level of convergence achieved.

The order of the errors (i.e., p) in both calculations are the same because the dominant term for the fully developed flow is the viscous term which is always second order accurate. “First-order upwind” refers to the discretization of the convective terms, which are zero for fully developed flow. Thus, there should be no difference between the first-order and second-order upwind schemes in the fully-developed region. However, the two schemes would not be equivalent in the developing flow region, where the convective terms are non-zero and the second-order scheme is expected to be superior.

