## Rigid body vs fluid

### Rigid body

- only one point is needed in order to track the dynamics of the object (e.g. COM)
- dynamics are solution to ODEs (ordinary differential equations)
- the dynamics are given by

\[
\begin{align*}
\text{position} & : \vec{X} \\
\text{orientation} & : \theta, \phi, \psi \\
\text{velocity} & : \vec{U}_{\text{com}} \\
\text{rotational velocity} & : \vec{\Omega}
\end{align*}
\]

### Fluid

- need to track all particles
- the fluid is a continuum medium, hence definition of fields (e.g. velocity field, temperature field, etc)
- dynamics are solution to PDEs (partial differential equations)
  - e.g. we solve for \( \vec{U}(\vec{x}, t) \) at every point instead of \( \vec{U}_{\text{com}}(t) \)

between parentheses is the number of unknowns
Reynolds number

Definition: \[ Re = \frac{UL}{\nu} \]

\( U \) = velocity scale
\( L \) = length scale
\( \nu \) = kinematic viscosity

Dimensions:

\[
\begin{align*}
[U] &= m/s \\
[L] &= m \\
[\nu] &= m^2/s
\end{align*}
\]

Re is dimensionless!
Reynolds number

\[ Re = \frac{UL}{\nu} \]

Illustration of parameters for flow past a cylinder
**Reynolds number**

- What are the typical values of Re number for different flow regimes?

\[ Re = \frac{\text{fluid inertia}}{\text{viscous forces}} \]

- e.g. 1: Jane walking leisurely in the auditorium

\[
\begin{align*}
V_{\text{ref}} & \sim 1 \text{ m/s} \\
L_{\text{ref}} & \sim 0.5 \text{ m} \\
\nu_{\text{air}} & \sim 10^{-5} \text{ m}^2/\text{s}
\end{align*}
\]

\[ Re \sim 5 \times 10^4 \]

- pretty turbulent regime! If you imagine smoke behind her, you’ll see swirls and eddies, signature of turbulent flow.
Reynolds number

What are the typical values of Re number for different flow regimes?

\[ Re = \frac{\text{fluid inertia}}{\text{viscous forces}} \]

* e.g. 2: a 10µm bead moving at a speed of 15cm/s in water

\[
\begin{align*}
V_{ref} &= 0.15 \, m/s \\
L_{ref} &= 10^{-5} \, m \\
\nu_{water} &= 10^{-6} \, m^2/s
\end{align*}
\]

\( \left\{ \right\} \quad Re = 1.5 \sim 1 \)

* a micro-bead @ high speed! This is borderline. Typical bacteria swim @ a couple body lengths per second, e.g. 20µm/s
Which flow has a higher Reynolds number?

- 3 containers filled with different fluids
- an incoming jet of ink is injected via a pipette

\[ Re_a < Re_b < Re_c \]

- visually, we can say that fluid ‘a’ is thicker or ‘stickier’ and the jet cannot push ‘hard’ enough
- fluid ‘c’ is able to penetrate further since it is not decelerated or damped by the ambient fluid
Mental picture of the Reynolds number

Stirring coffee vs Stirring honey

(*) admire the art! *)

once you stop stirring coffee, it keeps rotating. The moving fluid has enough inertia (from stirring with a spoon) to overcome the viscous forces. If you are not convinced, go to Starbucks in Collegetown, order a Frappucino and try it out ;-)

* once you stop stirring honey, it stops rotating immediately. The viscous forces are way higher than the inertia imparted to the honey.
Survey questions

- List the fluid variables needed to describe its state
  \[ \rho, \ T, \ \vec{V} \]

- List the fluid parameters that distinguish air from water
  \[ \rho, \ \mu \]

- Fluid equations
  - Conservation of mass (incompressible)
    \[ \nabla \cdot \vec{u} = 0 \]
  - Conservation of momentum
    \[ \rho \left[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = -\nabla p + \mu \nabla^2 \vec{u} \]