

MAE4230/5230

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# Intermediate Fluid Dynamics

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# Lecture 19

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- **Classical Aerodynamics**

# Aerodynamics

Given its geometry and the velocity, what is the force on an object moving in a fluid?

a reasonable question, but surprisingly difficult to answer.

# Governing Equations

Navier-Stokes equations for incompressible flows:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}$$
$$\nabla \cdot \mathbf{u} = 0$$

Boundary condition (no-slip) (wing kinematics):

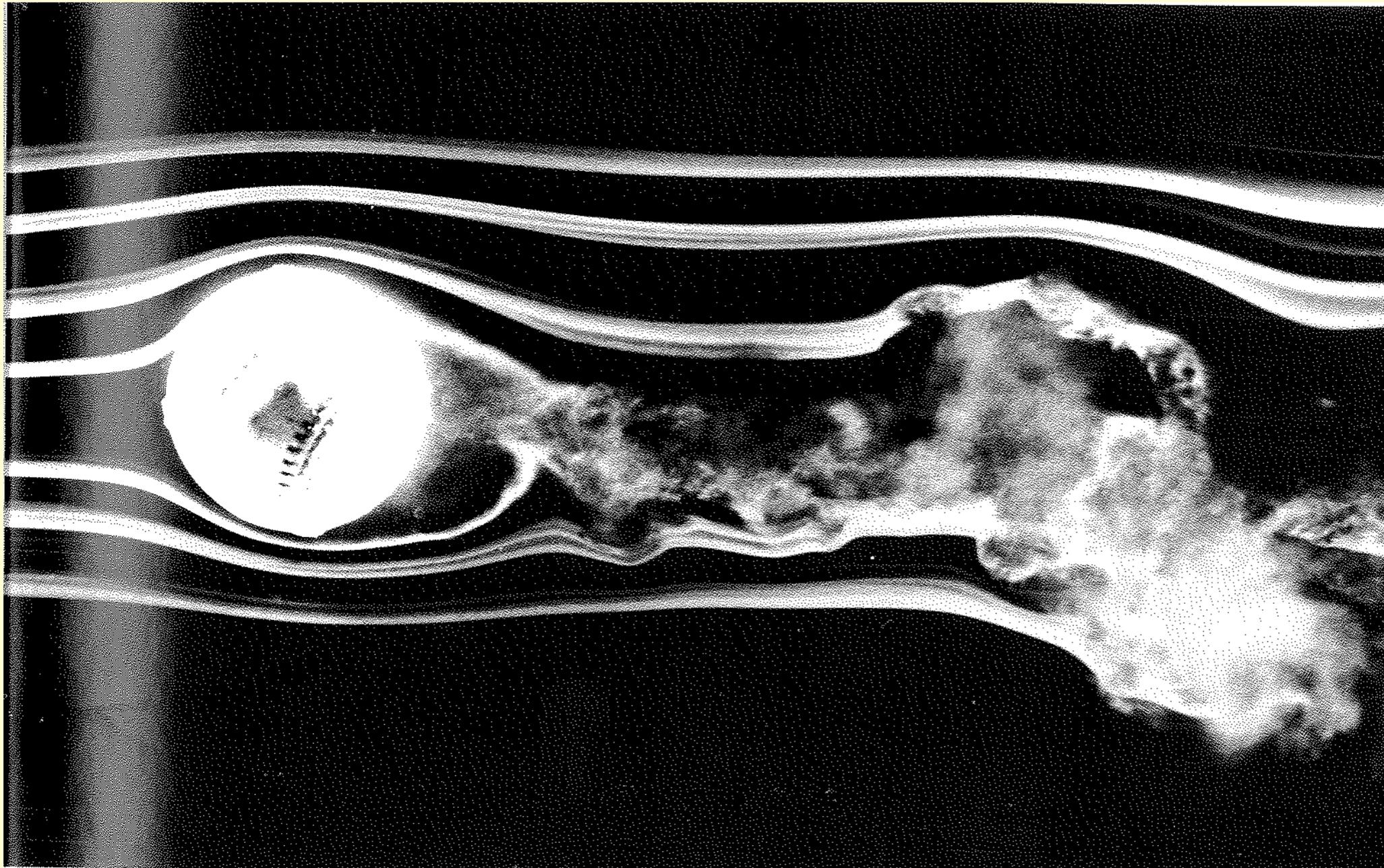
$$\mathbf{u}_b = \mathbf{v}_b$$

Dynamics of the solid object coupled to the fluid:

$$m \frac{d\mathbf{v}_b}{dt} = \mathbf{F}_{fluid} + \mathbf{F}_{ext}$$

# Direction of the force, without solving equations

What is force on the ball, and which direction does it spin?

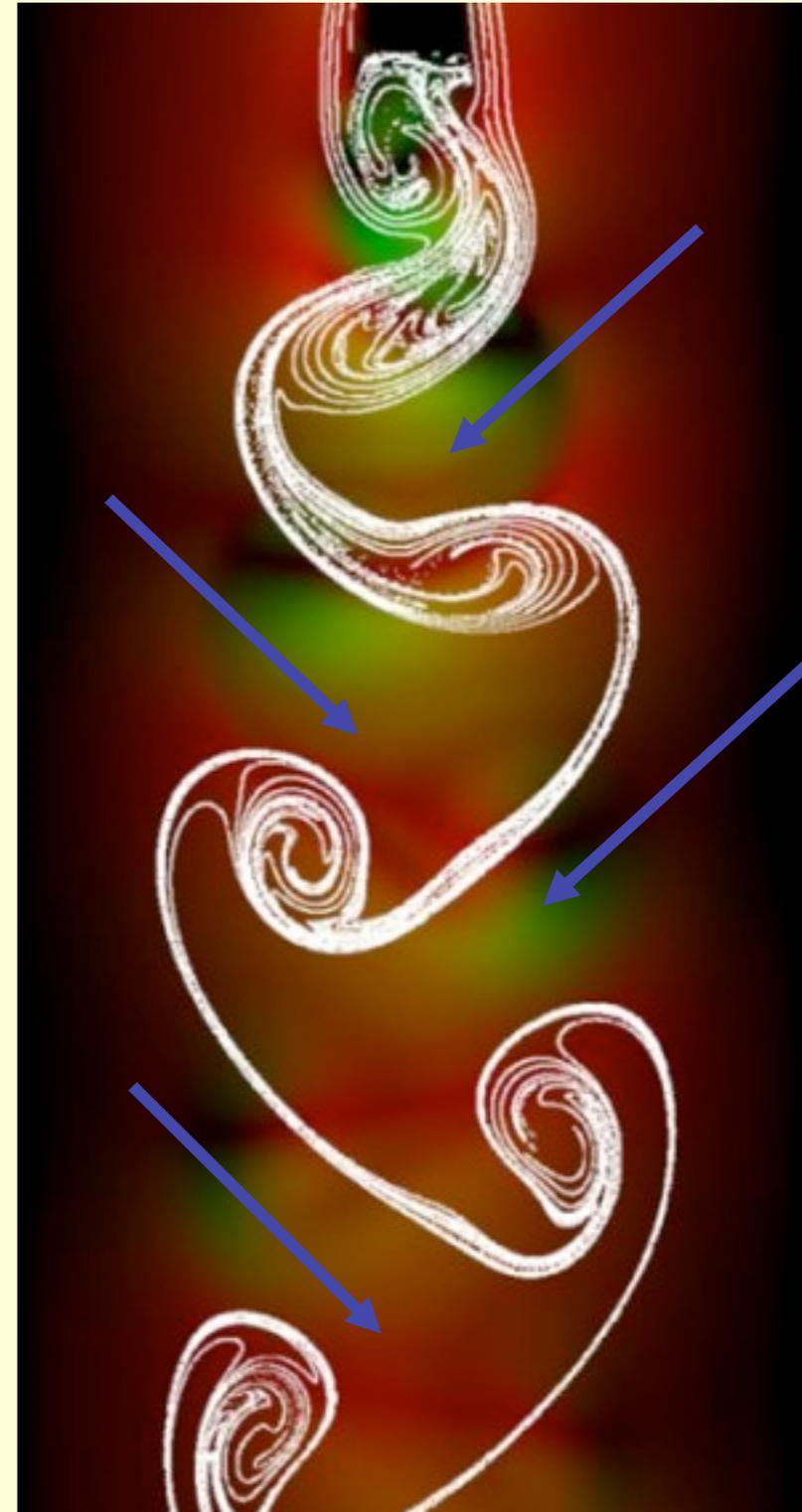
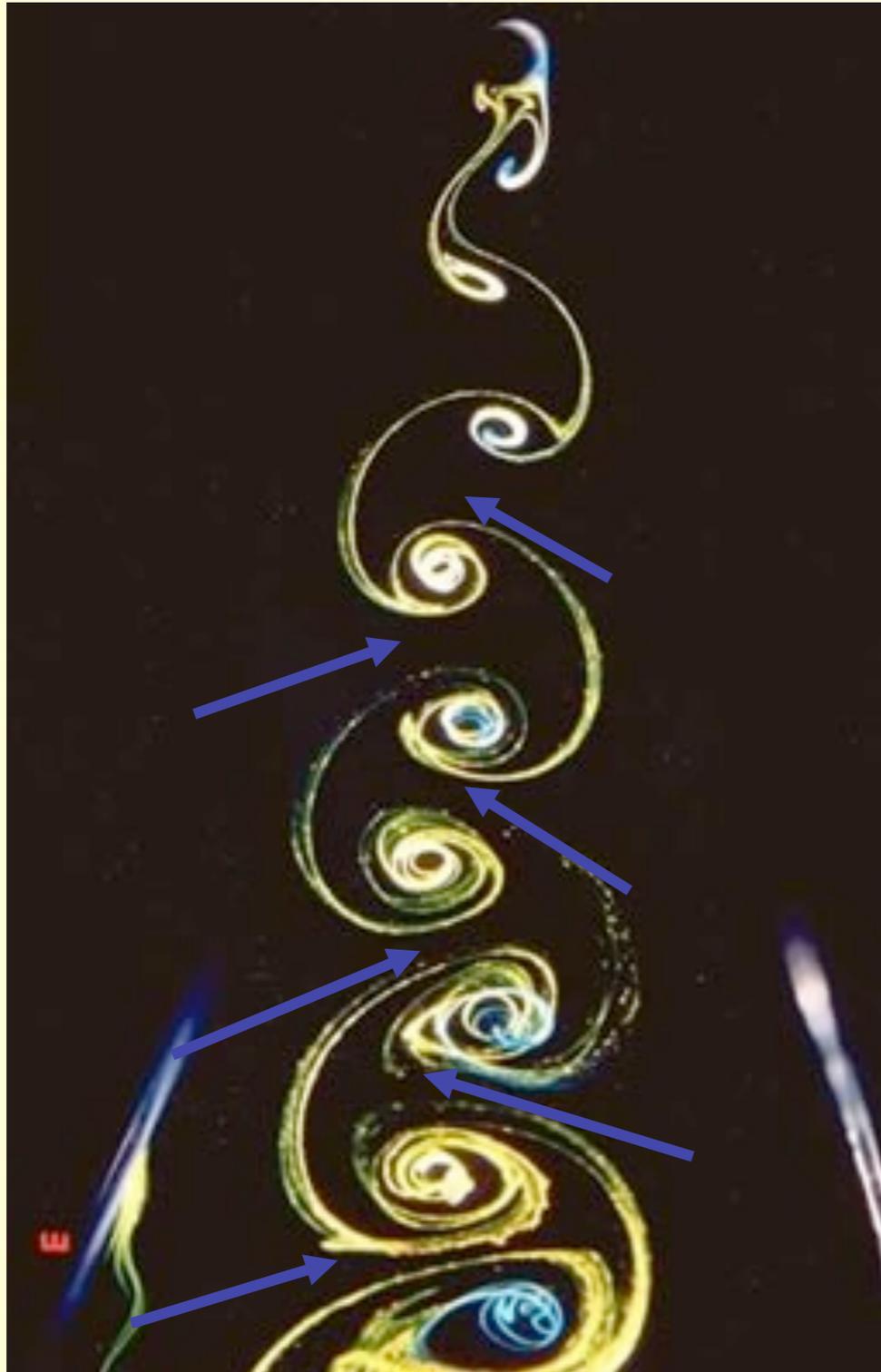


# Wake Signature of Thrust and Drag

- most agree that there is a lift on the ball, but there is confusion about **which direction does the ball turn.**

# Wake Signature of Thrust

## Von Karman and reversed Von Karman Wake



# Definition of Lift and Drag

## Lift

force component orthogonal  
to the wing velocity relative to flow at infinity

## drag

force component anti-parallel  
to the wing velocity relative to flow at infinity

# Remarks

- steady motion does not imply steady force
- at high  $Re$ , there is LIFT in addition to drag
- when the angle of attack  $\alpha$  is small, the rate of change of lift with respect to  $\alpha$  is much greater than that with drag, thus, as the angle of attack is increased, the lift-to-drag ratio can be significantly higher than one.

Stokes drag

Slender body theory

Attached flow

Classical Airfoil theory

Kutta Joukowski Theory  
for **LIFT**

Re 

Separated flow

Kirchhoff-Rayleigh  
theory for **DRAG**

# How to estimate lift to drag ratio

- define the lift to drag ratio during a steady gliding
- the best paper airplane we saw in the class had about a lift-to-drag ratio of 6

# Typical Lift to Drag Ratio

- airfoil: 10 -100
- the lift to drag ratio of high performance sailplane wing ~ 200
- the lift to drag ratio of sailplane ~50

# The Theoretical Estimate of Aerodynamic Lift

$$F_L = C_L (\rho U^2 L)$$

$$C_L = 2\pi \sin \alpha$$