MAE 4230/5230 Homework 8 Prof. Wang

Due Date: April 13, 2011

1) Bernoulli's law (again)

Read Chapter 1 in D. J. Acheson, and re-derive (on your own, no need to hand in)

a) the Euler equation
b) the Bernoulli streamline theorem
c) Bernoulli theorem for irrotational flow.

2) Rotating bucket of water

An ideal fluid is rotating under gravity g with constant angular velocity Ω. We wish to find the surface of a uniformly rotating bucket of water. By 'Bernoulli', $p/\rho + \frac{1}{2}u^2 + gz$, is constant, so the constant pressure surface are $z = c - \frac{\Omega^2}{2g}(x^2 + y^2)$. But this means that the surface of a rotating bucket of water is at its highest in the middle. What is wrong?

Write down Euler equation in component form and integrate them directly to find pressure p and hence to obtain the correct shape for the free surface.

3) Vorticity of a flow under constant rotation

A fluid undergoing solid body rotation Ω, show that the vorticity of the fluid is twice the rotational rate, $\omega = 2\Omega$, in three ways:

a) using the definition of the vorticity that relates its magnitude to the angular rotational rate of two orthogonal lines,
b) expressing the circulation in terms of the vorticity via Stokes's theorem,
c) applying directly the vector curl to the velocity field.

4) Vorticity of flow around a line vortex

We saw in the class that the velocity field generated by a vortex line, $U_\theta(r) = 1/r$, has zero vorticity everywhere except for the origin. Confirm this by applying the vector curl to $U_\theta(r)$ in the cylindrical coordinates.

5) Vorticity of rotating Couette flow

The flow inside a rotating Couette cell is given by $U_\theta(r) = Ar + B/r$, which is the superposition of the flow around a line vortex and the flow due solid rotation.

a) Determine the vorticity of this flow, based on answers to questions 3 and 4.

b) If the inner cylinder at $r = r_1$ is rotating at an angular rotational rate of Ω_1, and the outer cylinder at $r = r_2$ is rotating at an angular rotational rate of Ω_2, one can show that $A = \frac{\Omega_2 r_2^2 - \Omega_1 r_1^2}{r_2^2 - r_1^2}$. Interpret the meaning of A in light of our discussions on vorticity.

6) Streamlines

Consider the unsteady flow $u = u_0, v = kt$, where u_0 and k are positive constants. Show that the streamlines are straight lines, and sketch them at two different times. Also show that any fluid particle follows a parabolic path as time proceeds.

7) Vector Potential and Stream Function

Read chapter 4.2, 4.3 in Acheson to review the definition of the vector potential ϕ and the stream function Ψ. Show that both ϕ and Ψ satisfy the Laplace equation.