MAE 4230/5230 Homework 6 Prof. Wang

Due Date: March 9, 2011

1)Reading

Read Chapter 8 Boundary Layers in D. J. Acheson (DJA)

2) Boundary Layer equations in non-dimensional forms

Rewrite the exact 2D equations of motion in terms of the non-dimensional and scaled variables

$$x' = \frac{x}{L}, \quad y' = \frac{y}{\mathrm{Re}^{-1/2} L}, \; u' = \frac{u}{U_0}, \; v' = \frac{v}{\mathrm{Re}^{-1/2} U_0}, \; p' = \frac{p}{\rho U_0^2},$$

where $\mathrm{Re} = U_0 L / \nu$. By taking the limit $\mathrm{Re} \to \infty$ with fixed $u', \partial u' / \partial x'$, etc.. derive the boundary layer equations in their non-dimensional and scaled form in steady state:

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = -\frac{\partial p'}{\partial x'} + \frac{\partial^2 u'}{\partial y'^2},$$

$$0 = -\frac{\partial p'}{\partial y'}, \; \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 .$$

3) Matlab solution of the similarity equation

For flow past a plate, the similarity solution $f(\eta)$ is governed by

$f''' + ff'' = 0,$ with boundary conditions $f(0) = f'(0) = 0, f'(\infty) = 1.$

Solve $f(\eta)$ numerical using Matlab and plot your solution.

[Hint: you can define $g(\eta) = f'(\eta), h(\eta) = g'(\eta)$ to reduce the 3rd order differential equation into three first order differential equations.]

4) Drag on a plate

The flow velocity in the boundary layer is given by

$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x},$ where $\Psi = (2\nu U x)^{1/2} f(\eta),$ with $\eta = \frac{y}{(2\nu x / U)^{1/2}}$.

a) Find the shear stress along the plate.
b) Using the results from part a) and also the numerical solution from question 3 to determine the drag on a plate of length L.