MAE 4230/5230 Homework 5

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Laminar Pipe Flow

Consider developing flow in a pipe of length $L = 8$ m, diameter $D = 0.2$ m, $\rho = 1$ kg/m^3, $\mu = 2 \times 10^{-3}$ kg/m s, and entrance velocity $u_{\text{in}} = 1$ m/s. This case is solved in the laminar pipe flow tutorial at *https://confluence.cornell.edu/x/6YQaBQ*. Go through the tutorial. You can download the mesh at the top of the geometry step and skip the geometry and mesh steps. Use FLUENT with the "second-order upwind" scheme for momentum to solve for the flowfield on meshes of 100×10, 100×20 and 100×40 (axial points $\times$ radial points).

1. Plot the axial velocity profiles at the exit obtained from the three meshes. Also, plot the corresponding velocity profile obtained from fully-developed pipe analysis. Indicate the equation you used to generate this profile. In all, you should have four curves in a single plot. Use a legend to identify the various curves. Axial velocity u should be on the abscissa and r on the ordinate.

   Hint: In FLUENT, you can write out the data in any "XY" plot to a file by selecting the "Write to File" option in the *Solution XY Plot* menu. Then click on *Write* and enter a filename. You can strip the headers and footers in this file and read this into MATLAB as column data using the *load* function in MATLAB.

2. Calculate the shear stress τ_{xy} at the wall in the fully-developed region for the three meshes. Calculate the corresponding value from fully-developed pipe analysis. For each mesh, calculate the % error relative to the analytical value. Include your results as a table:

Mesh	τ_{xy}	% error

3. At the exit of the pipe where the flow is fully-developed, we can define the error in the centerline velocity as

$$\epsilon = \frac{|u_c - u_{\text{exact}}|}{u_{\text{exact}}}$$

where u_c is the centerline value from FLUENT and u_{exact} is the corresponding exact (analytical) value. We expect the error to take the form

$$\epsilon = K \Delta r^p$$

where the coefficient K and power p depend upon the order of accuracy of the discretization. Note that Δr is the grid spacing in the radial direction. Using MATLAB, perform a linear least squares fit of

$$\ln \epsilon = \ln K + p \ln \Delta r$$

to obtain the coefficients p and K. Plot ϵ vs. Δr (using symbols) on a log-log plot. Add a line corresponding to the least-squares fit to this plot.

4. Let's see how p changes when using a first-order accurate discretization. In FLUENT, use "first-order upwind" scheme for momentum to solve for the flowfield on the three meshes. Repeat the calculation of coefficients p and K as above. Add this ϵ vs. Δr data (using symbols) to the above log-log plot. Add a line corresponding to the least-squares fit to this plot. In all, you should have four curves on this plot (two each for second- and first-order discretization). Make sure you include an appropriate legend in the figure.

   Contrast the value of p obtained in the two cases and briefly explain your results (2-3 sentences).

   Hint: To interpret your results, you should keep in mind that the first or second-order upwind discretization applies only to the inertia (advection) terms in the momentum equation. The discretization of the viscous terms is always second-order accurate.