

1) Reading

Read Chapter 2 in D. J. Acheson (DJA)

2) Flow under gravity down an inclined plane

This is a preparation for problem 3. Repeat the analysis in Acheson (pg 38-40). Examine the solution and comment on the relation of this solution to 1d pipe flow.

3) Problem 2.4 in Acheson

Two incompressible viscous fluids of the same density ρ flow, one on top of the other, down an inclined plane making an angle α with the horizontal. Their viscosities are μ_1 and μ_2 , the lower fluid is of depth h_1 and the upper fluid is of depth h_2 . Show that

$$u_1 = [(h_1 + h_2)y - \frac{1}{2}y^2] \frac{g \sin \alpha}{\nu_1},$$

so that the velocity of the fluid $u_1(y)$ is dependent on the depth h_2 , but not the viscosity, of the upper fluid. Why is this.

4) Problem 2.6 in Acheson

Viscous fluid flows between two rigid boundaries $y = 0$, $y = h$, the lower boundary moving in the x -direction with constant speed U , the upper boundary being at rest. The boundaries are porous, and the vertical velocity v is v_0 at each one, v_0 being a given constant (so that there is an imposed flow across the system). Show that the resulting flow is

$$u = U \left(\frac{e^{-v_0 y / \nu} - e^{-v_0 h / \nu}}{1 - e^{-v_0 h / \nu}} \right), \quad v = -v_0.$$

Show that the horizontal velocity profile $u(y)$ has a very thin boundary layer adjacent to $y = 0$.