MAE 4230/5230 Homework 2 Prof. Wang

Due Date: Feburary 9, 2011

1)Taylor's lecture

Watch G. I. Taylor's film and read the associated notes on low Reynolds number flow at http://mit.edu/hml/ncfmf.html

Describe one phenomenon that you find striking and are likely to remember long after the course is over.

2) A falling sphere in a viscous fluid at zero Reynolds number

At very low Reynolds numbers, the drag, F_d , on a sphere is given the by the Stokes law: $F_d = 6\pi\mu UR$. Where μ is the fluid dynamic viscosity, R is the radius of the sphere, and U is the velocity of the sphere relative to the far field flow. Calculate the settling velocity of a falling sphere of mass m in a highly viscous fluid.

Note that this is a method for measuring viscosity. In his famous oil droplet experiment, Milikan determined the electric charge on an oil droplet by measuring the the settling speed of the droplet in a electric field.

3) Pipe flow

Consider unidirectional flow in a cylinder. Assume that the flow is fully developed and steady.

a) Using the Navier-Stokes equation, derive the general solution for the velocity profile in the pipe.

b) Graph the velocity and shear stress profile as a function of radial position and comment on the shape of the profiles. What is the shear stress along the wall? What is the total friction the wall exerts on the fluid? How is the total friction related to the pressure difference at two ends of the pipe?

4) Potential flow

A potential flow is an irrotational incompressible flow.

a) By irrotational flow, we mean that $\nabla \times u = 0$. Using this, together with the mass conservation, show that the velocity potential ϕ satisfies the Laplace equation.

b) For a one dimensional flow, solve for the velocity potential that satisfies the boundary condition $\phi(0) = 1$ and $\phi(2) = 10$. Determine the velocity field.

c) What can you say about the general property of the velocity field of 1d potential flow.