Homework 1 Solutions

1.

a)

$$Re = \frac{UL}{\upsilon}$$

$$U = 10\frac{mi}{h} = 22.5\frac{m}{s}\ , \upsilon_{air}\ \approx 15\ X\ 10^{-6}\frac{m^2}{s}, L \sim .1m\ (Flag\ Pole\ Diameter)$$

$$Re \approx \frac{\left(22.5\frac{m}{s}\right)\cdot(.1m)}{15\ X\ 10^{-6}\frac{m^2}{s}} = 150\ X\ 10^3$$

b)

Michael Phelp's World Record Free-Style, March 27, 2007: $\frac{200\ m}{103.86\ s} = 1.93\frac{m}{s}$. So a typical swimmer's velocity is of the order $U \sim 1\ m/s$. $L \sim 1\ m$ (frontal profile span).

$$\upsilon_{water}\ \approx 1\ X\ 10^{-6}\frac{m^2}{s}$$

$$Re \approx \frac{\left(1\frac{m}{s}\right)\cdot(1m)}{1\ X\ 10^{-6}\frac{m^2}{s}} = 1\ X\ 10^6$$

c)

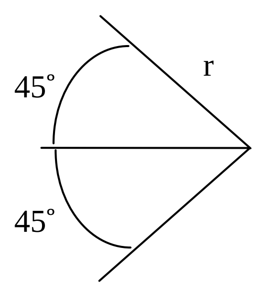
Dragonfly flapping frequency $\approx 40\ Hz$ (Wang)

Estimate wing length$\sim 5\ cm$:

Estimate wing chord length (Length Scale): $\sim 1\ cm$

$$\upsilon_{air}\ \approx 15\ X\ 10^{-6}\frac{m^2}{s}$$

Assuming fluttering takes place between 45˚ above/below horizontal:



The total distance one wing traverses in a flutter (up and down) motion is $\pi \cdot r$ or:

$$\pi \cdot (5X10^{-2}m) = .157m$$

$$\therefore \text{Wing Velocity} \approx \frac{distance}{cycle} \cdot \frac{cycle}{time} = .157\, m \cdot 40\, Hz = 6.28\, m/s$$

$$Re \approx \frac{\left(6.28\, \frac{m}{s}\right) \cdot (1\, X\, 10^{-2}\, m)}{15\, X\, 10^{-6}\, \frac{m^2}{s}} = 4.19\, X\, 10^3$$

This agrees with the expected Re of between 3000 and 6000 (Wang).

2.

$Re \sim O(10^{-3})$: Stokes Flow, red-blood cell flow in a capillary
$Re \sim O(10^{2})$: Flow past a moving beetle
$Re \sim O(10^{5})$ A person walking at a comfortable pace

3.

Consider the example discussed in class about stirring of coffee vs. stirring honey. If we stir honey for a time and remove the stirring spoon, the honey quickly settles to rest. This suggests that the viscous forces in this system are high compared to the inertial forces, hence a relatively low Reynolds number. However, when we stop stirring the coffee, we see the surface rotates long after the spoon has been removed. This suggest that the inertial forces are large compared with the viscous forces and thus, suggests a relatively large Reynolds number.

4.

If we think of steady, developed, uni-directional flow between two plates, one that is stationary, and one that is moving at a uniform velocity, we can show from the Navier-Stokes equation that the fluid velocity profile for this system is linear, that is:

$$u(y) = Ay + B$$

Where y is the spatial dimension perpendicular to the plates, and A and B are constants determined by the boundary conditions. The shear stress for the system is given by:

$$\tau = \mu \frac{\partial u}{\partial y} = \mu A$$

Therefore, we see that the shear stress is a constant for this type of flow. The shear stress is directly proportional to the velocity gradient in y, that is, a shear stress exists because there is a difference in velocity at different y distances in the flow. The constant of proportionality is the dynamic viscosity.

The kinematic viscosity is then conveniently defined as:

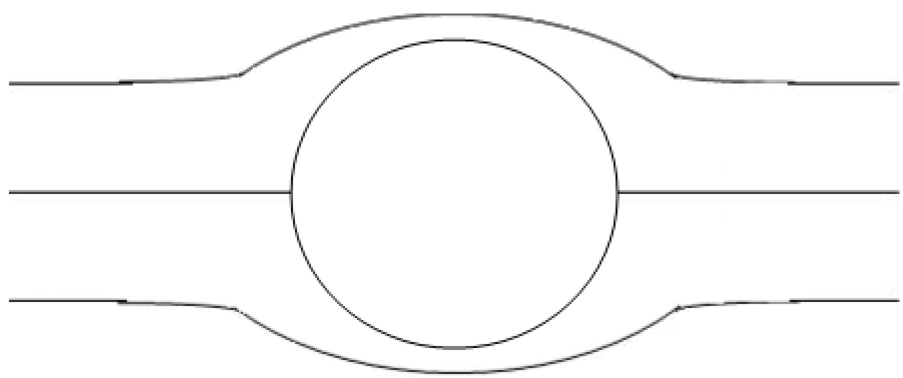
$$\upsilon = \mu/\rho$$

And is also referred to as the fluid **momentum diffusivity**.

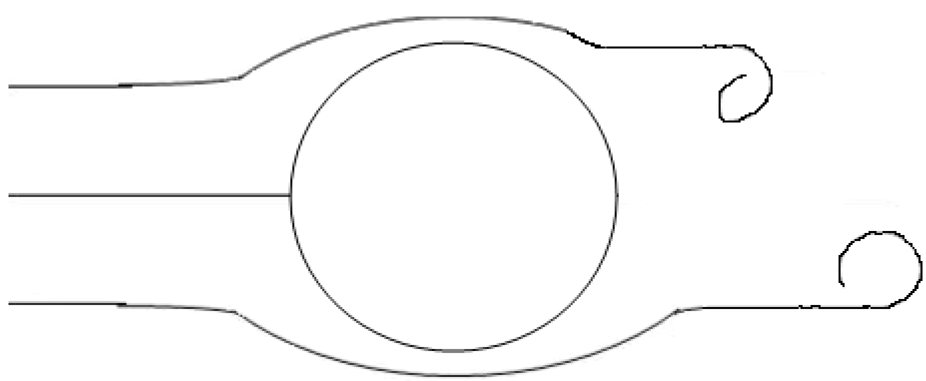
$$\upsilon_{air} \approx 15\ X\ 10^{-6}\frac{m^2}{s}$$
$$\upsilon_{water} \approx 1\ X\ 10^{-6}\frac{m^2}{s}$$
$$\upsilon_{glycerin} \approx 5\ X\ 10^{-4}\frac{m^2}{s}$$
$$\upsilon_{motor\ oil} \approx 5\ X\ 10^{-4}\frac{m^2}{s}$$
$$\upsilon_{honey} \approx 7.3\ X\ 10^{-5}\frac{m^2}{s}$$

5.

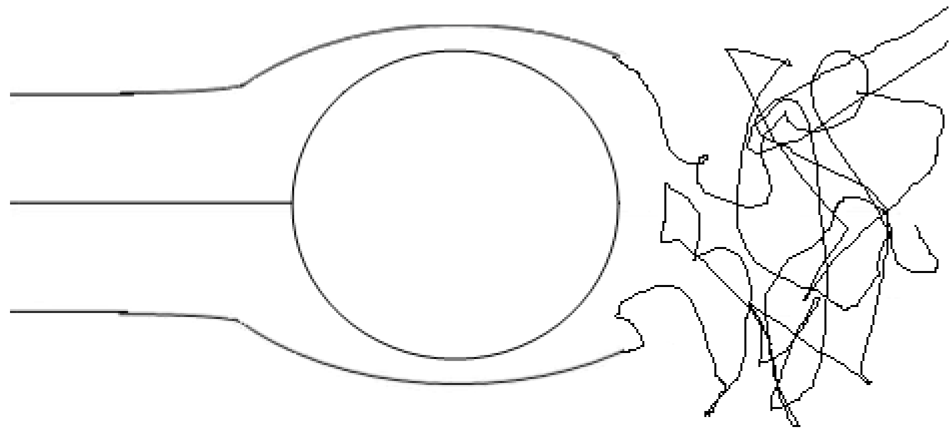
$$Re \ll 1$$



$$Re \sim 100$$



$Re \sim 10^6$



Lack of symmetry in wake suggests lift force in addition to drag force.

6.

Ball travels to the left; the wake is seen moving downward, indicating a downward fluid momentum transport. In other words, the fluid is pushed down by the ball. By Newton's 3rd law, the ball must experience an upward force from the fluid. This suggests the pressure at the bottom of the ball is higher than at the top. By Bernoulli's law, the velocity on the bottom side must be lower than on the top side. If we consider the tangential component of the velocity on the bottom side of the ball, we observe that this velocity must be smaller than the velocity of the golf ball's center of mass. The only way for this to be true is for the component of the tangential velocity due to the rotation of the ball to be in the reverse direction of the ball's travel. Therefore, if the ball is traveling to the left, this velocity component must be to the right which immediately says the ball is spinning **counter-clockwise**.