MAE 4230/5230 Homework 10 Prof. Wang

Due Date: April 27, 2011

1) Read Chapter 4 in Acheson

2) Stream function ψ

The velocity of an incompressible flow, $\vec{U}(\vec{x},t)$, is divergence free, i.e., $\nabla \cdot \vec{U}(\vec{x},t) = 0$. Therefore, we can express $\vec{U}(\vec{x},t)$ in terms of a vector potential, $\psi(\vec{x},t)\hat{z}$ by writing $U(\vec{x},t) = \nabla \times \psi(\vec{x},t)\hat{z}$.

a) show that in 2D, the velocity components are given by $U_x = \frac{\partial \psi}{\partial y}, U_y = -\frac{\partial \psi}{\partial x}$.

b) show that ψ is constant along the streamline. For this reason, ψ is referred to as the stream function.

3) Complex potential $W(z) = \phi(x,y) + i\psi(x,y)$**,** $z = x + iy$

The complex potential associated with a potential flow, $U(\vec{x},t)$, is defined to be $W(z) = \phi(x,y) + i\psi(x,y)$, where ϕ is the vector potential of the velocity field, i.e., $U(\vec{x},t) = \nabla\phi$, and ψ is the stream function, $U(\vec{x},t) = \nabla \times \psi(\vec{x},t)\hat{z}$. The complex velocity, $U_x - iU_y = \frac{dW}{dz}$.

For potential flow past a cylinder of radius a, $W(z) = U\left(z + \frac{a^2}{z}\right)$,

a) find the expressions for $\phi(x,y)$ and $\psi(x,y)$.

b) show that $r = a$ is a streamline. This implies that the normal velocity at the cylindrical surface must be zero.

c) alternatively, you can show that the normal velocity at $r = a$ is zero by finding the velocity field with $U(\vec{x},t) = \nabla \times \psi(\vec{x},t)\hat{z}$, and examine its normal component to the cylinder.

d) plot the streamlines of this flow using Matlab.

e) find the slip velocity U_θ and show that it is not zero everywhere.

4) Conformal mapping

Consider the mapping from z to ξ : $\xi = \left(z + \frac{a^2}{z} \right)$,

a) show that this function maps the circle of radius $r = a$ in the z-plane into a line that lies from $-2a$ to $2a$ on the real axis in the ξ-plane.

b) show that it maps a circle of radius $r > a$ into an ellipse.

5) Lift and Drag Coefficient

a) Draw a diagram to define the lift (F_L)and drag (F_D) on an airfoil moving at velocity U. The fluid has a density ρ and the wing area is A, find an appropriate force scale for the wing. In 2D, the dimension of the wing is given by its length L. Find the corresponding force scale and its dimension.

b) Sketch the flow around an airfoil and describe the flow condition at the trailing edge.

c) For a steady translating airfoil at a fixed angle of attack, show that its efficiency, the inverse of the work required to transport a unit weight a unit distance is given by the lift to drag ratio.